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Comment on "The Phenomenology of a Nonstandard Higgs Boson in $W_L W_L$ Scattering"

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Abstract

We show that in Composite Higgs models, the coupling of the Higgs resonance to a pair of W bosons is weaker than the corresponding Standard Model coupling, provided the Higgs arises from electroweak doublets only. This is partly due to the effects of the nonlinear realization of the chiral symmetries at the compositeness scale.

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In a recent paper [1], Koulovassilopoulos and Chivukula presented a Composite Higgs model based on the chiral symmetry breaking pattern $SU(4)/Sp(4)$, where the coupling of the isoscalar "Higgs" resonance to a pair of W bosons (and consequently its partial decay width to WW and ZZ) was smaller than its value in the Standard Model. In this Comment we show that, to lowest order in chiral perturbation theory, this is true in all Composite Higgs models [2,3], provided the isoscalar resonance arises from electroweak doublets only. This fact is well known in the case of *linear* models of elementary scalars. The new element in the proof that follows is that the effects of the nonlinear realization of the chiral symmetry reduce the strength of the coupling of the Higgs particle to WW even further.

In Composite Higgs models, the Higgs arises as a pseudo-Goldstone boson of the spontaneous breakdown of the chiral symmetries of ultrafermions. We denote the chiral symmetry group by G. At some scale f, the strong ultracolor dynamics causes the group G to spontaneously break down to a subgroup H. The Goldstone boson manifold G/H can be parametrized by the field

$$\Sigma = \exp\left(\frac{2\,i\,\Pi^\alpha \mathcal{X}^\alpha}{f}\right) \tag{1}$$

where Π^α are the Goldstone boson fields and $\mathcal{X}^\alpha$ the broken generators normalized so that $\mathrm{Tr}\,(\mathcal{X}^a\mathcal{X}^b) = \frac{1}{2}\delta^{ab}$. Under $g \in H$, Σ transforms as

$$\Sigma \rightarrow g\,\Sigma\,g^\dagger. \tag{2}$$

Since $SU(2)_W \times U(1)_Y \subseteq H$, the interactions of the electroweak gauge bosons with Π^α are described to lowest order in momentum by a chiral lagrangian

$$\mathcal{L}_\Sigma = \frac{f^2}{4}\mathrm{Tr}\,\left(D_\mu\Sigma^\dagger\, D^\mu\Sigma\right) \tag{3}$$

where the covariant derivative is

$$D_\mu\Sigma = \partial_\mu\Sigma + ig[S^a,\Sigma]W_\mu^a + ig'[Y,\Sigma]\,\mathcal{B}_\mu \tag{4}$$

where S^a, Y belong to the algebra H and generate $SU(2)_L$, $U(1)_Y$ transformations respectively.

Among the Π^α, there are four fields σ, w_1, w_2, w_3, which *by assumption* transform in the fundamental representation of $SU(2)_L$, *i.e.* they form an electroweak doublet

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} w_1 + iw_2 \\ \sigma + iw_3 \end{pmatrix} \tag{5}$$

In order to illustrate more clearly the effects of the nonlinear realization, we assume that only one (composite) doublet is responsible for electroweak symmetry breaking. In what follows, we shall set to zero all the other Goldstone bosons since they do not affect our argument. Furthermore, we can ignore also hypercharge (*i.e.* set $g' = 0$). The generalization to $g' \neq 0$ is straightforward.

The dynamics of the vacuum alignment is responsible for giving Σ a vacuum expectation value $\langle\Sigma\rangle = \exp(2iG\langle\sigma\rangle/f)$, where G is the corresponding generator. The term in the lagrangian (3) which describes the interactions of the Higgs with the gauge boson is

$$\mathcal{L}_{WW\sigma} = \frac{g^2}{8} M^2(\sigma)\, W^a_\mu W^{a\mu} \tag{6}$$

where

$$\frac{1}{2} M^2(\sigma) = -f^2 \mathrm{Tr}\left[S^3, e^{-\frac{2i\sigma G}{f}}\right]\left[S^3, e^{\frac{2i\sigma G}{f}}\right] \tag{7}$$

Note that $M^2(\sigma)$ is positive definite due to the hermiticity of S^3, G. We have also set the exact Goldstone bosons w_i to zero, by going over to the unitary gauge. By defining the shifted field $\sigma = \langle\sigma\rangle + H$, the lagrangian above expanded in terms of H gives

$$\mathcal{L}_{WW\sigma} = \frac{1}{2} M_W^2 W^a_\mu W^{a\mu} + \frac{g^2 v}{4} \xi H W^a_\mu W^{a\mu} + \mathcal{O}(H^2) \tag{8}$$

where

$$M_W^2 = \frac{1}{4} g^2 M^2(\langle\sigma\rangle) \tag{9}$$

$$v = M(\langle\sigma\rangle) \tag{10}$$

and

$$\xi = M'(\langle\sigma\rangle) \tag{11}$$

with the prime denoting differentiation. Thus, ξ parametrizes the strength of the Higgs coupling to a pair of W bosons. The Standard Model has $\xi = 1$; hence we have to show that $M'(\sigma) \leq 1$.

Consider first the limit $f \to \infty$. Then

$$\frac{1}{2} M^2(\sigma) = -4\sigma^2 \mathrm{Tr}\,[S^3, G][S^3, G] + \mathcal{O}\left(1/f^2\right) \tag{12}$$

In order to evaluate this trace we have to make use of the assumption that σ belongs to the doublet (5). This is tantamount to the relation

$$[S^3, G] = i\frac{1}{2}\mathcal{X}^3 \tag{13}$$

where $\mathcal{X}^3$ is the broken generator that corresponds to the w_3 Goldstone boson, correctly normalized $\mathrm{Tr}\,(\mathcal{X}^3)^2 = 1/2$. It thus follows that

$$\mathrm{Tr}\,[S^3, G][S^3, G] = -\frac{1}{8} \quad . \tag{14}$$

Consequently, from eq. (12) it follows that $M(\sigma) = \sigma$ and thus $\xi = 1$. This is the limit where the composite Higgs models reduce to the standard model. Notice however, that the relation (14) holds even in the general case of finite f which we now consider.

To evaluate the expression (7), it is convenient to express S^3 as a sum of eigenvectors of G:

$$S^3 = \sum_i b_i E_i \tag{15}$$

where the E_i are defined by

$$[G, E_i] = \lambda_i E_i. \tag{16}$$

So λ_i are the (real) eigenvalues of G in the adjoint representation. If we normalize the E_i's so that

$$\mathrm{Tr}\,E_i^\dagger E_j = \frac{1}{2}\delta_{ij} \tag{17}$$

then the normalization of S^3 implies that

$$\sum_i |b_i|^2 = 1 \tag{18}$$

while eq. (14) implies that

$$\sum_i \lambda_i^2\,|b_i|^2 = \frac{1}{4} \quad . \tag{19}$$

We are now in a position to evaluate eq. (7). Let $U = \exp(2i\sigma G/f)$. Then

$$\mathrm{Tr}\,[S^3, U^\dagger][S^3, U] = 2\,\mathrm{Tr}\,(S^3 U^\dagger S^3 U) - 1 \quad . \tag{20}$$

By expanding U, and using the Baker-Campbell-Hausdorff formula and eqs. (15), (16) we obtain

$$\begin{aligned} U^\dagger S^3 U &= S^3 - \frac{2i\sigma}{f}[G, S^3] + \frac{1}{2!}\left(-\frac{2i\sigma}{f}\right)^2 \Big[G, [G, S^3]\Big] + \ldots \\ &= \sum_i b_i E_i\, e^{-2i\sigma\lambda_i/f} \end{aligned} \tag{21}$$

Consequently,

$$\mathrm{Tr}\,(S^3U^\dagger S^3U)=\frac{1}{2}\sum_i |b_i|^2\, e^{-2i\sigma\lambda_i/f} \tag{22}$$

Since $\mathrm{tr}\,S^3U^\dagger S^3U$ is real, it follows that

$$\mathrm{tr}\,S^3U^\dagger S^3U=\frac{1}{2}\sum_i |b_i|^2\cos\frac{2\sigma\lambda_i}{f} \tag{23}$$

and, by virtue of (18),

$$\begin{aligned} 2\;\mathrm{tr}\,S^3U^\dagger S^3U-1 &= \sum_i |b_i|^2\left(\cos\frac{2\sigma\lambda_i}{f}-1\right) \\ &= -2\,\sum_i |b_i|^2\sin^2\frac{\sigma\lambda_i}{f} \end{aligned} \tag{24}$$

Hence

$$M^2(\sigma)=4\,f^2\,\sum_i |b_i|^2\sin^2\frac{\sigma\lambda_i}{f} \tag{25}$$

To show $M'(\sigma)\le 1$, it suffices to show that $(d/d\sigma)M^2(\sigma)\le 2M(\sigma)$ (for $M>0$).

$$\begin{aligned} \frac{d}{d\sigma}M^2(\sigma) &= 8f^2\sum_i |b_i|^2\,\sin\frac{\sigma\lambda_i}{f}\cos\frac{\sigma\lambda_i}{f}\cdot\frac{\lambda_i}{f} \\ &\le 8f\sum_i |b_i|^2|\lambda_i|\left|\sin\frac{\sigma\lambda_i}{f}\right| \\ &\le 8f\cdot\sqrt{\sum_i \lambda_i^2|b_i|^2}\,\sqrt{\sum_i |b_i|^2\sin^2\frac{\sigma\lambda_i}{f}} \\ &= 2\,M(\sigma) \end{aligned} \tag{26}$$

where we have used (19). The penultimate step is obvious if we define "vectors" $\vec{A},\vec{B}$ with components $A_i=|\lambda_i b_i|, B_i=|b_i\sin(\sigma\lambda_i/f)|$ and employ the fact that $\vec{A}\cdot\vec{B}\le|\vec{A}||\vec{B}|$. This completes the proof that, to lowest order in chiral perturbation theory, the parameter ξ cannot exceed its Standard Model value of 1, provided the Higgs belongs to an electroweak doublet representation. In some Composite Higgs models [2], including the one presented in ref. [1], the Higgs field appears as a linear combination of fields that belong to two different doublets, one of which is usually taken to be fundamental. In this case ξ is reduced even further, in the same way as in linear two-doublet models. For example, in the model of ref. [1], ξ can be written in terms of mixing angles α and β as

$$\xi=\sin\alpha\sin\beta+M'(\sigma)\cos\alpha\cos\beta \tag{27}$$

which obviously is smaller than one since $M'(\sigma) \leq 1$.

We thus conclude that, if the Higgs resonance arises from electroweak doublets only, then its coupling to a pair of W bosons is smaller than the corresponding Standard Model coupling. In order to obtain $\xi > 1$, the Higgs boson must be part of a larger representation of $SU(2)_L$. This possibility has been emphasized recently by Chivukula, Dugan and Golden [4].

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